

LAST TIME: $4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots\right) = \pi$

Why does the sum equal π ?

Taylor series for $\arctan(x)$:

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

set $x=1$: $\arctan(1) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

$$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots\right)$$

Code from last time:

sum = 0

Do[sum += (-1)^i / (2i+1), {i, 0, 100}]

outputs an approximation of $\pi \rightarrow 4 * \text{sum}$

↑
number of terms
is 101

MODULES IN MATHEMATICA

A module is a block of code that has a name and maybe some parameters.

↙ parameter: number of terms

Example:

computeSum[n_] := Module[{sum = 0},

Do[sum += (-1)^i / (2i+1), {i, 0, n-1}];

4 * sum ← return value or output

]

EXAMPLE: $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$

see the "functions and pi" notebook on the course web page.