PELL SEQUENCE: 0, 1, 2, 5, 12, 29, 70, ...

$$\boxed{P_0 = 0, \quad P_1 = 1, \quad P_n = 2P_{n-1} + P_{n-2} \quad \text{for } n \geq 2}$$

Interesting application: $\sqrt{2} \approx 1.4142\ldots$

$\frac{1}{1} = 1, \quad \frac{3}{2} = 1.5, \quad \frac{7}{5} = 1.4, \quad \frac{17}{12} = 2.41\overline{6}, \quad \frac{41}{29} = 1.4138\ldots$

no integer solutions $\rightarrow$ $\frac{x}{y} = \sqrt{2}$ if and only if $x^2 - 2y^2 = 0$.

If $\boxed{x^2 - 2y^2 = \pm 1}$ Pell's equation then $\frac{x}{y} \approx \sqrt{2}$.

$3^2 - 2(2)^2 = 1$

$7^2 - 2(5)^2 = -1$

etc.

every 3rd Pell num. | cube of Pell numbers | Pell numbers

	0		0			0
$n=1 \rightarrow$	5	=	8 · 1	−	3 ·	1
$n=2 \rightarrow$	70	=	8 · 8	+	3 ·	2
$n=3 \rightarrow$	985	=	8 · 125	−	3 ·	5
$n=4 \rightarrow$	13 860	=	8 · 1728	+	3 ·	12
$n=5 \rightarrow$	195 025	=	8 · 24 389	−	3 ·	29
$n=6 \rightarrow$	2 744 210	=	8 · 343 000	+	3 ·	70

Looks like: $\boxed{P_{3n} = 8(P_n)^3 + (-1)^n 3 P_n}$ $\leftarrow$ The 3n identity

Question: Suppose we think $P_{3n} = a(P_n)^3 + b P_n$ for even n.

How could we find a and b?

Question: Is there a 5n identity?

try something like:

$$P_{5n} = a(P_n)^5 + b(P_n)^3 + c\,P_n$$