Pell Sequence: 0, 1, 2, 5, 12, 29, 70, ...

$P_0 = 0,\ P_1 = 1,\quad P_n = 2P_{n-1} + P_{n-2}$ for $n \geq 2$

Approximating $\sqrt{2} \approx 1.4142...$

$\frac{1}{1} = 1,\ \frac{3}{2} = 1.5,\ \frac{7}{5} = 1.4,\ \frac{17}{12} = 1.41\overline{6},\ \frac{41}{29} = 1.41379...$

no integer solutions $\rightarrow$ $\frac{x}{y} = \sqrt{2}$ if and only if $x^2 - 2y^2 = 0$

Pell's Equation

If $\boxed{x^2 - 2y^2 = \pm 1}$, then $\frac{x}{y} \approx \sqrt{2}$.

$3^2 - 2(2)^2 = 1$

$7^2 - 2(5)^2 = -1$

etc.

every third Pell num. | cube of Pell nums. | Pell nums.

| | | | | | | |
|---|---|---|---|---|---|---|
| | | 0 | | 0 | | 0 |
| $n=1$ | $\rightarrow$ | 5 | = | $8 \cdot 1$ | − | $3 \cdot 1$ |
| $n=2$ | $\rightarrow$ | 70 | = | $8 \cdot 8$ | + | $3 \cdot 2$ |
| $n=3$ | $\rightarrow$ | 985 | = | $8 \cdot 125$ | − | $3 \cdot 5$ |
| $n=4$ | $\rightarrow$ | 13 860 | = | $8 \cdot 1728$ | + | $3 \cdot 12$ |
| $n=5$ | $\rightarrow$ | 195 025 | = | $8 \cdot 24\,389$ | − | $3 \cdot 29$ |
| $n=6$ | $\rightarrow$ | 2 744 210 | = | $8 \cdot 343\,000$ | + | $3 \cdot 70$ |

It looks like $P_{3n} = 8 \cdot (P_n)^3 + (-1)^n\, 3\, P_n$ 3n identity

Suppose we think $P_{3n} = a(P_n)^3 + b\,P_n$ for even n.

How would we find a and b?

Is there a $5n$ identity?

$$P_{5n} = a(P_n)^5 + b(P_n)^3 + c\,P_n$$

solve for $a, b, c$

even $n$?