CASSINI's
IDENTITY:

$$
F_{n-1} F_{n+1}-F_{n}^{2}=(-1)^{n}
$$

proof:

$$
\begin{aligned}
& {\left[\begin{array}{ll}
1 & 1 \\
1 & 0
\end{array}\right]\left[\begin{array}{l}
1 \\
1
\end{array} 0\right]=\left[\begin{array}{ll}
2 & 1 \\
1 & 1
\end{array}\right]} \\
& {\left[\begin{array}{ll}
2 & 1 \\
1 & 1
\end{array}\right]\left[\begin{array}{ll}
1 & 1 \\
1 & 0
\end{array}\right]=\left[\begin{array}{ll}
3 & 2 \\
2 & 1
\end{array}\right]} \\
& {\left[\begin{array}{ll}
3 & 2 \\
2 & 1
\end{array}\right]\left[\begin{array}{ll}
1 & 1 \\
1 & 0
\end{array}\right]=\left[\begin{array}{ll}
5 & 3 \\
3 & 2
\end{array}\right]} \\
& {\left[\begin{array}{ll}
F_{n+1} & F_{n} \\
F_{n} & F_{n-1}
\end{array}\right]\left[\begin{array}{ll}
1 & 1 \\
1 & 0
\end{array}\right]=\left[\begin{array}{ll}
F_{n+1}+F_{n} & F_{n+1} \\
F_{n}+F_{n-1} & F_{n}
\end{array}\right]} \\
& \vdots \\
& =\left[\begin{array}{ll}
F_{n+2} & F_{n+1} \\
F_{n+1} & F_{n}
\end{array}\right]
\end{aligned}
$$

for integers $\left[\begin{array}{ll}1 & 1 \\ 1 & \vdots\end{array}\right]^{n}=\left[\begin{array}{ll}F_{n+1} & F_{n} \\ F_{n} & F_{n-1}\end{array}\right]$
Take determinants:

$$
F_{n-1} F_{n+1}-F_{n}^{2}=\operatorname{det}\left[\begin{array}{cc}
F_{n+1} & F_{1} \\
F_{n} & F_{n-1}
\end{array}\right]=\operatorname{det}\left[\begin{array}{ll}
1 & 1 \\
1 & 0
\end{array}\right]^{n}=\left(\operatorname{det}\left[\begin{array}{l}
1 \\
1 \\
1
\end{array}\right]\right]^{n}=(-1)^{n}
$$

Thus: $\quad F_{n-1} F_{n+1}-F_{n}^{2}=(-1)^{n}$

Generalize: $\quad F_{n-2} F_{n+2}-F_{n}^{2}=$ ?

$$
\begin{gathered}
F_{n-3} F_{n+3}-F_{n}^{2}=? \\
\vdots \\
F_{n-k} F_{n+k}-F_{n}^{2}=?
\end{gathered}
$$

