Fibonacci Polynomial Identities

look for an identity of the form:

$$F_{3n} = a F_n^3 + b F_n^2 + c F_n$$

We have found that $F_{3n} = 5F_n^3 - 3F_n$

holds for $n = 1, 3, 5, 7, 9$

Fibonacci 3n Identity: $F_{3n} = 5F_n^3 + (-1)^n 3F_n$

Fibonacci 5n Identity: $F_{5n} = 25F_n^5 + (-1)^n 25F_n^3 + 5F_n$