

FIBONACCI POLYNOMIAL IDENTITIES

$F_{2n} = ?$

$$F_{3n} = 5 F_n^3 + (-1)^n 3 F_n$$

$F_{4n} = ?$

$$F_{5n} = 25 F_n^5 + (-1)^n 25 F_n^3 + 5 F_n$$

$F_{6n} = ?$

$$F_{7n} = 125 F_n^7 + (-1)^n 175 F_n^5 + 70 F_n^3 + (-1)^n 7 F_n$$

⋮

Observations? Questions?

Generalize the Fibonacci numbers:

$$F_0 = 0, \quad F_1 = 1, \quad \text{and} \quad F_n = F_{n-1} + F_{n-2} \quad \text{for } n > 1$$

① We could change the initial values

We could define a generalized Fibonacci sequence
"gibonacci"

$$G_0 = a, \quad G_1 = b \quad \text{and} \quad G_n = G_{n-1} + G_{n-2} \quad \text{for } n > 1.$$

↑
any integers we likeLucas sequence: $a = 2, \quad b = 1$

$$L_0 = 2, \quad L_1 = 1, \quad \text{and} \quad L_n = L_{n-1} + L_{n-2} \quad \text{for } n > 1$$

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, ...

② We could change the recurrence.

Next time...