

GENERALIZED FIBONACCI NUMBERS

Today: change the recurrence:

$$G_0 = a, G_1 = b, \text{ and } G_n = r \cdot G_{n-1} + s \cdot G_{n-2}$$

for integers $n > 1$ and some a, b, r, s .

Define the Pell Sequence: $a = 0, b = 1, r = 2, s = 1$

$$P_0 = 0, P_1 = 1, P_n = 2P_{n-1} + P_{n-2}$$

Sequence: 0, 1, 2, 5, 12, 29, 70, 169, 408, ...

Application: approximating $\sqrt{2} = 1.414213\dots$

Start with $\frac{x}{y} = \sqrt{2}$ ← has no integer solutions

$$\begin{aligned} \downarrow x &= y\sqrt{2} \\ x^2 &= 2y^2 \\ x^2 - 2y^2 &= 0 \end{aligned}$$

Replace 0 with ± 1 : $x^2 - 2y^2 = \pm 1$ ← Pell's equation

Solutions:	$x=1$	$x=3$	$x=7$	$x=17$
	$y=1$	$y=2$	$y=5$	$y=12$

$\frac{x}{y} = \frac{1}{1} = 1.$	$\frac{x}{y} = \frac{3}{2} = 1.5$	$\frac{x}{y} = \frac{7}{5} = 1.4$	$\frac{x}{y} = \frac{17}{12} = 1.41\dots$
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sequence: $\frac{1}{1}, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \frac{99}{70}, \dots \rightarrow \sqrt{2}$

SILVER RATIO: $1 + \sqrt{2}$

