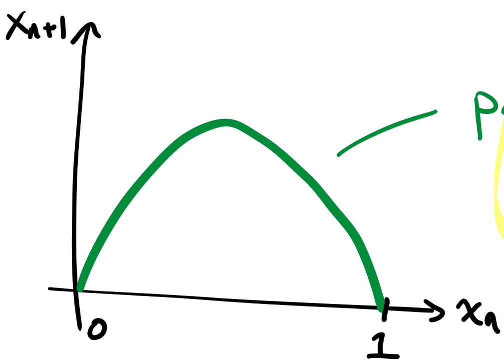
# The Logistic Map

Sequence of relative population sizes between 0 and 1:

$x_0, x_1, x_2, x_3, \ldots$

Concept: If $x_n$ is close to 0, then so is $x_{n+1}$.
If $x_n$ is close to 1, then $x_{n+1}$ is close to 0.
If $x_n$ is near $\frac{1}{2}$, then $x_{n+1}$ is big.




parabola?

$$\boxed{x_{n+1} = r \cdot x_n(1-x_n)}$$

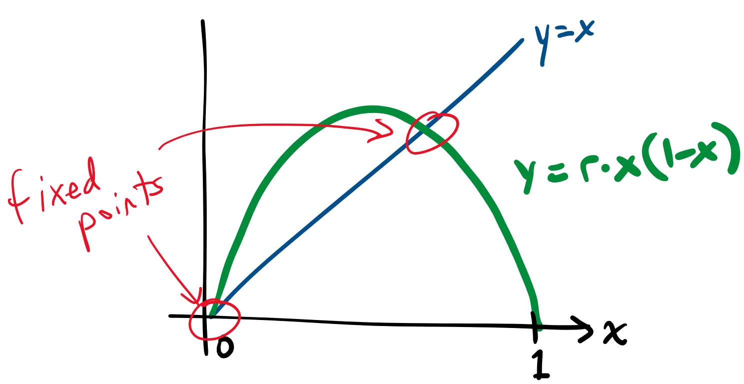
$r$ is some constant

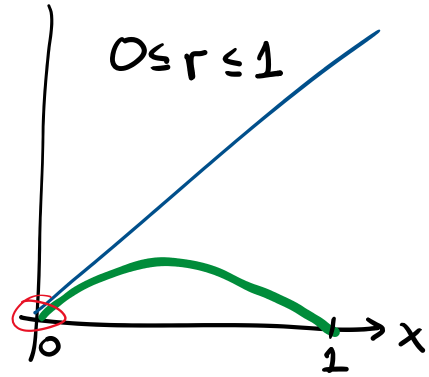
NOTE: This recurrence is a discrete version of the logistic equation $\frac{df}{dx} = f(x)\big(1-f(x)\big)$.

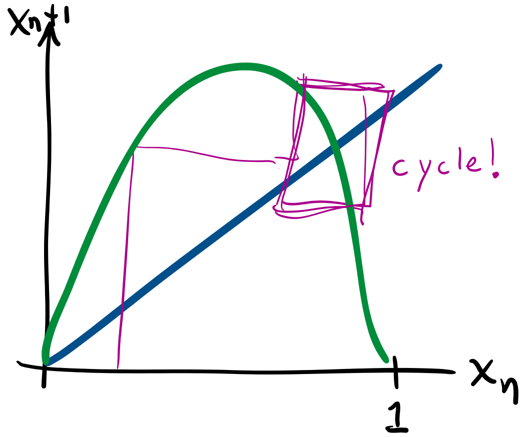
**FIXED POINT:** a value $x^*$ such that

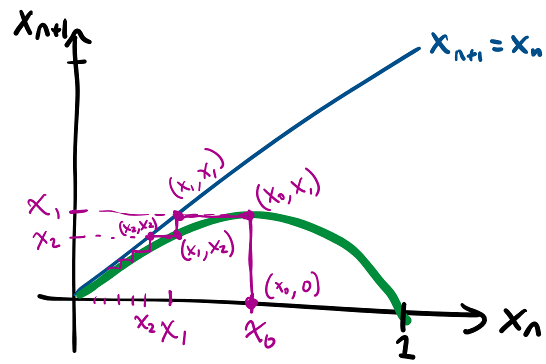
$$x^* = r \cdot x^*(1-x^*)$$

$x^* = 0$ is always a fixed point, for any $r$.




# Summary:

If $0 \le r \le 1$, then $x_n$ converges to 0.

If $1 < r \le 3$, then $x_n$ converges to the other fixed point.

If $r$ is somewhat larger than 3, then $x_n$ approaches a stable oscillation between two values.