The Riemann Zeta Function

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots$$

example: $\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$

1. What did Riemann hypothesize in his 1859 paper?
   The nontrivial zeros have real part 1/2.

2. How do the zeta zeros relate to the prime numbers?
   You can use them to "predict" the prime numbers, or predict the harmonics of the prime counting function.
   There is a version of the prime counting function that can be approximated very well by a sum of functions that depend on the zeros of the zeta function.

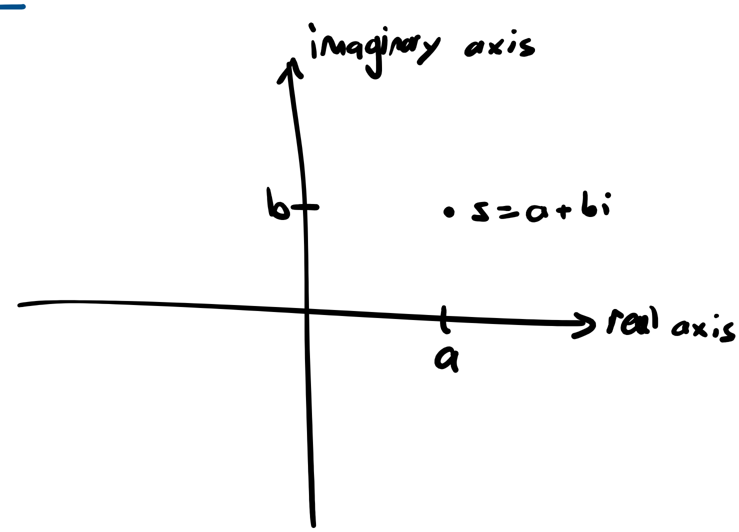
Complex Plane:

$s = a + bi$

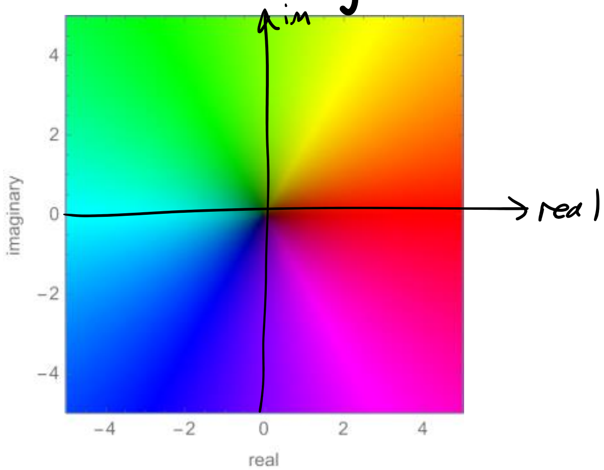
a: real part

b: imaginary part

$i^2 = -1$



Domain Coloring



ζ(s) domain coloring: