MATH 242 — 25 February 2026

What virtues have you acquired as a result of doing mathematics?

From Monday:

$$\underset{F_0}{0}, \underset{F_1}{1}, \underset{F_2, \dots}{1}, 2, 3, 5, 8, 13, 21, 34, \underset{F_{10}}{55}, \dots$$

Conjecture: The Fibonacci numbers satisfy $\underbrace{F_{n-1}F_{n+1} - F_n^2 = (-1)^n}_{\text{identity}}$.

proof:

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}$$

Why? If $n=0$, $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} F_2 & F_1 \\ F_1 & F_0 \end{bmatrix}$.

Induction:
$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{n+1} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} F_{n+1} + F_n & F_{n+1} + 0 \\ F_n + F_{n-1} & F_n + 0 \end{bmatrix}$$
$$= \begin{bmatrix} F_{n+2} & F_{n+1} \\ F_{n+1} & F_n \end{bmatrix}$$

take determinants

$$\left(\det \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)^n = \det\left(\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n\right) = \det \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}$$

$$(-1)^n = F_{n+1}F_{n-1} - F_n^2$$