$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55$$

F_0 F_1 F_3 $F_n = F_{n-1} + F_{n-2}$

QUESTION: Could there exist a Fibonacci identity of the form

$$F_{3n} = a F_n^3 + b F_n^2 + c F_n$$

for some constants a, b, c?

← polynomial like $ax^3 + bx^2 + cx$

Equations: try $n = 1$: $F_{3 \cdot 1} = a F_1^3 + b F_1^2 + c F_1$

$$2 = 1a + 1b + 1c$$

Found: $a = 5$, $b = 0$, $c = -3$ holds for $n \in \{1, 3, 5, 7, 9\}$

Conjecture: The Fibonacci numbers satisfy

$$F_{3n} = 5 F_n^3 + (-1)^n 3 F_n$$

for all ~~odd~~ positive integers n.

$5n$: $F_{5n} = 25 F_n^5 + (-1)^n 25 F_n^3 + 5 F_n$?