MATH 242 — 2 March 2026

Fibonacci Polynomial Identities

What did you observe?

What questions do you have?

Fibonacci Sequence: $F_0 = 0,\ F_1 = 1,\ F_n = F_{n-1} + F_{n-2}$

We could change: initial values (↑ today) or the recurrence. (↑ Wednesday)

Lucas Sequence: $L_0 = 2,\ L_1 = 1,\ L_n = L_{n-1} + L_{n-2}$

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, ...

Conjectures:

1. The Lucas numbers satisfy $\lim_{n\to\infty} \frac{L_{n+1}}{L_n} = \frac{1+\sqrt{5}}{2}$.

2. The Fibonacci and Lucas numbers satisfy $F_{n-1} + F_{n+1} = L_n$.

3. The Lucas numbers satisfy $\sum_{k=0}^{n} L_k = L_{n+2} - 1$.

4. The Fibonacci and Lucas numbers satisfy $F_n L_n = F_{2n}$.

5. The Lucas numbers satisfy $L_{2n} = L_n^2 + (-1)^{n+1} 2$.

6. The Fibonacci and Lucas numbers satisfy

$$L_{m+1} F_n + L_m F_{n-1} = L_{m+n}.$$