THE LOGISTIC MAP

$$f_r(x) = r \cdot x (1-x)$$

parameter: $0 \le r \le 4$
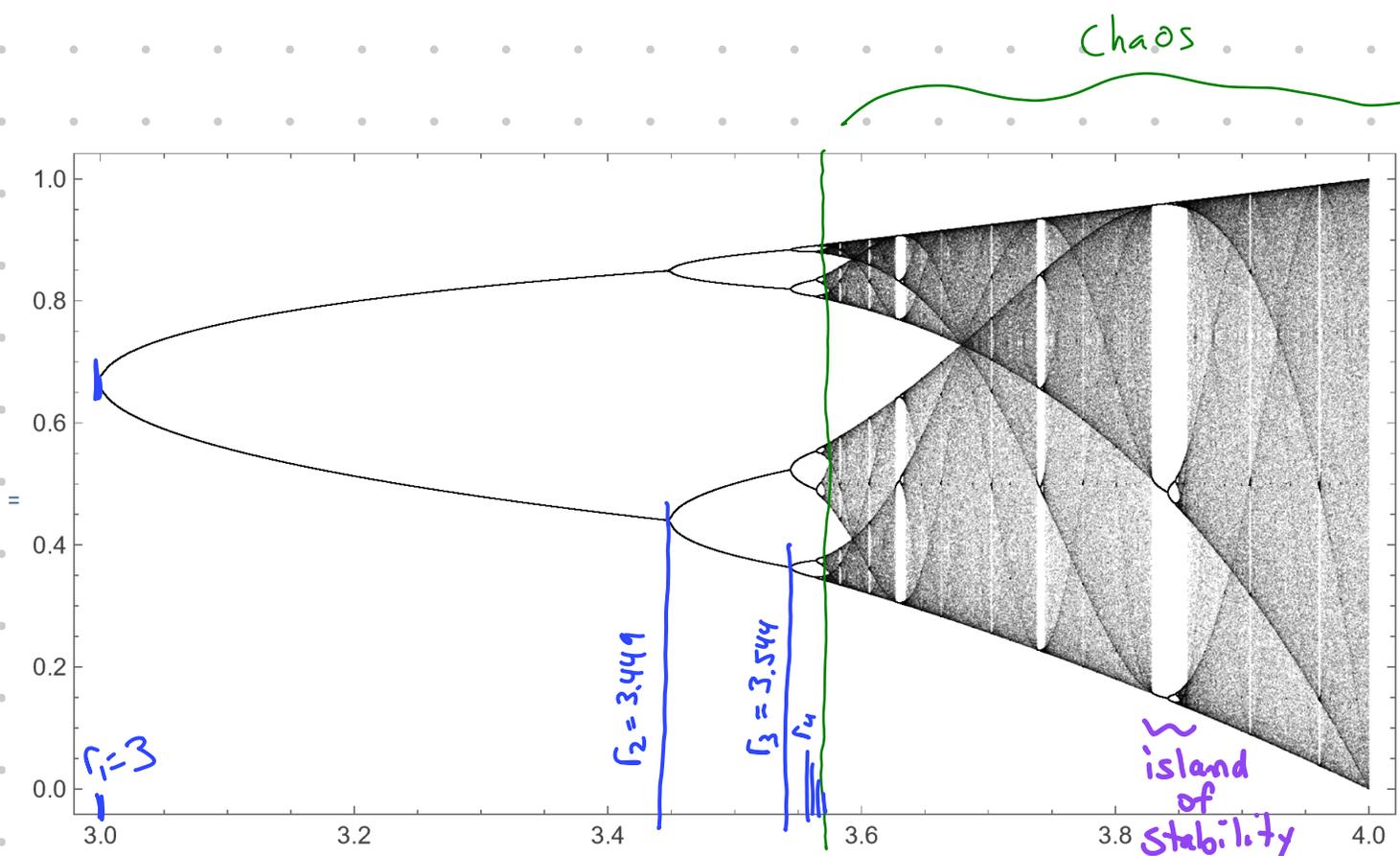
variable: $0 \le x \le 1$

RECALL:

- If $0 \le r \le 1$, then the trajectories converge to 0.
- If $1 < r < 3$, then the trajectories converge to $x^* = \frac{r-1}{r}$
- If $3 < r < 3.449$, then the trajectories oscillate between two values.
- If $3.449 < r < 3.544$, then the trajectories oscillate among four values.

- As r increases, we observe a sequence of period-doubling bifurcations.

BIFURCATION DIAGRAM

$x^* = \frac{r-1}{r}$

Chaos

$r_1 = 3$

$r_2 = 3.449$

$r_3 = 3.544$

r_4
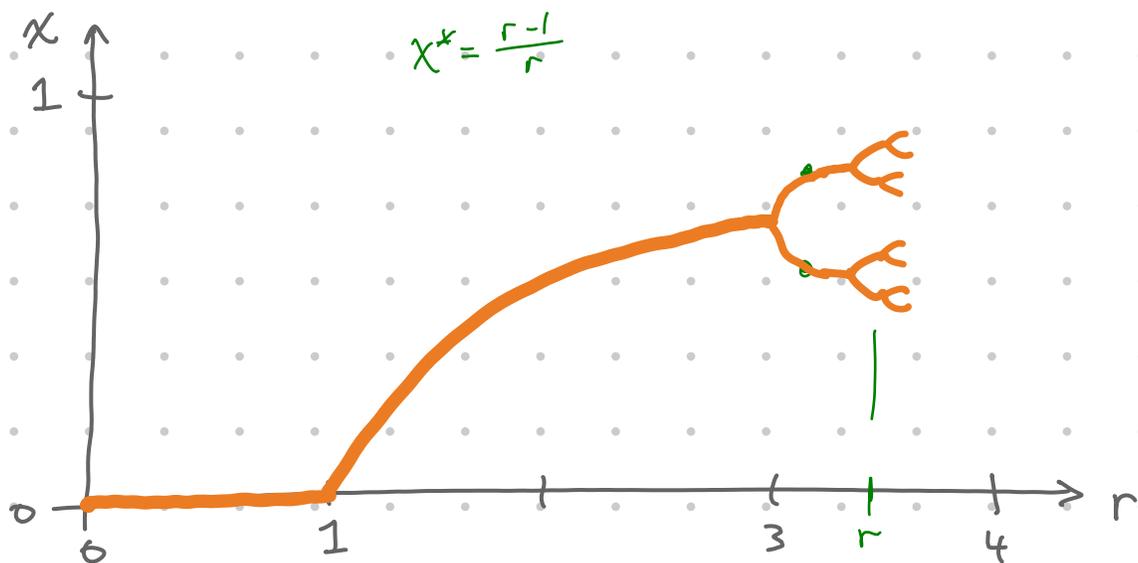
$r_\infty \approx 3.569995$

period-doubling cascade
2, 4, 8, 16, 32, 64, ...

onset of chaos

island of stability
3-cycles
6, 12, 24, ...