Polygon Triangulation Theorem

MATH 261 Computational Geometry

Complete the proof of the following theorem.

Theorem: Every triangulation of a polygon P with n vertices has $n-2$ triangles and $n-3$ diagonals.

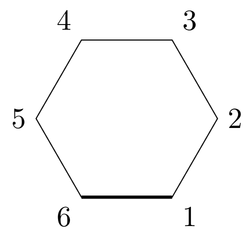
Proof (by induction):
First state the base case. Explain why it is true.

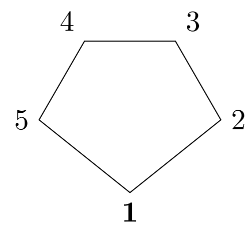
Inductive hypothesis: Let $n > 3$ be an integer, and assume the statement is true for all polygons with fewer than n vertices.
Now explain how the statement follows for a polygon with n vertices.

Edge Contraction Example

MATH 261 Computational Geometry

Let P_n be a convex polygon with vertices labeled 1 to n counterclockwise. Let $\mathcal{T}_n$ be the set of all triangulations of P_n, and let t_n be the number of elements of $\mathcal{T}_n$.
Define the map $\phi : \mathcal{T}_6 \to \mathcal{T}_5$ that contracts the edge $\{1, 6\}$ to the point 1. To illustrate this map, first draw all triangulations in $\mathcal{T}_6$ and $\mathcal{T}_5$. Then draw an arrow from each triangulation $T \in \mathcal{T}_6$ to $\phi(T)$.





Use your illustration to explain why

$$t_6 = \sum_{T \in \mathcal{T}_5} [\text{degree of vertex 1 in triangulation } T].$$

Then sum over all vertices of T to explain why

$$5 \cdot t_6 = 2 \cdot 7 \cdot t_5.$$

Now generalize. What equation relates t_{n+2} and t_{n+1}?