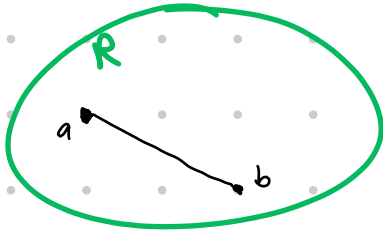
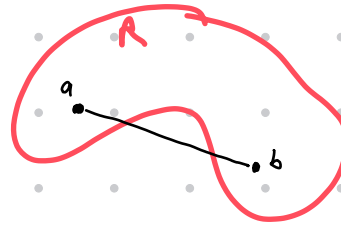


8 January 2025

A region  $R$  is **CONVEX** if, for any two points  $a$  and  $b$  in  $R$ , the line segment  $\overline{ab}$  is in  $R$ .

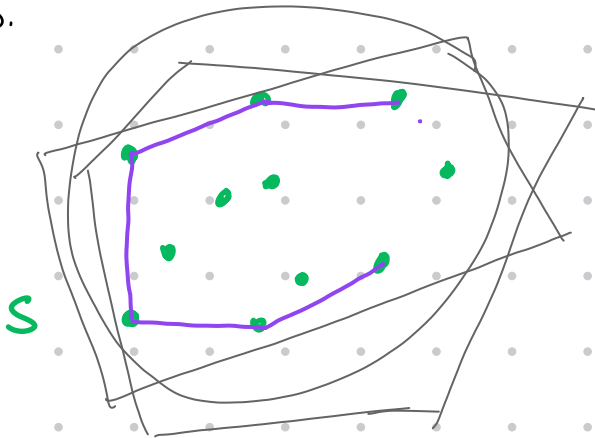


convex



not convex

Let  $S$  be a set of points. The **CONVEX HULL** of  $S$ , denoted  $\text{conv}(S)$ , is the intersection of all convex regions containing  $S$ .



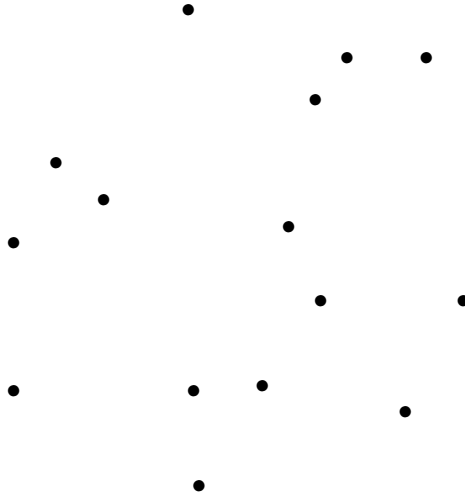


# Convex Hulls

MATH 261 Computational Geometry

1. Let  $S$  be a set of points in the plane. Given points  $a$  and  $b$  in  $S$ , how would you determine whether the segment  $ab$  is part of the convex hull  $\text{conv}(S)$ ?

Here is a sample set of points  $S$ :



2. Given the coordinates of all points in  $S$ , how would you program a computer to determine whether a particular pair of points in  $S$  are endpoints of an edge of  $\text{conv}(S)$ ?

Here is a sample set of points, specified by coordinates:

(0.9, 4.7)  
(1.1, 9.3)  
(6.6, 2.5)  
(8.2, 1.8)  
(6.8, 8.1)  
(4.7, 7.3)  
(3.8, 1.5)  
(5.0, 2.9)  
(2.6, 5.2)  
(5.9, 6.4)

3. Given the coordinates of all points in  $S$ , how would you program a computer to find *all* edges of  $\text{conv}(S)$ ?

4. How many operations would your algorithm require to find the convex hull of 10 points? ...of 100 points? ...of 1000 points?





# Convex Hull Incremental Algorithm

MATH 261 Computational Geometry

**Input:** a set  $S$  of  $n$  points in the plane, specified by  $xy$ -coordinates

**Output:** a list  $L$  of vertices of  $\text{conv}(S)$  in counterclockwise order

**Algorithm:**

1. Sort the points in  $S$  by their  $x$ -coordinates. Let the resulting list be denoted  $p_1, p_2, p_3, \dots, p_n$ .
2. Consider the first three points  $p_1, p_2, p_3$ . Let  $H_3$  be a list containing these points in counterclockwise order.

3. For  $k$  from 4 to  $n$ :

Consider  $H_{k-1}$  together with  $p_k$ . Remove interior points, and insert  $p_k$  into the list to form a new list  $H_k$ . Then  $H_k$  is the convex hull of the first  $k$  points, in counterclockwise order.

*How would you program a computer to do this?*

4. Let  $L = H_n$ .