Final Exam: Take-Home

Name: _____

Math 262: Probability Theory

Due Friday, December 13, at the final exam session (9am)

Take-Home Instructions:

1. *For this part of the exam, you may use your textbook, your notes, the course web site, and computing technology (such as R,* Mathematica*,* Wolfram Alpha*, or a calculator). If you use technology to compute something, indicate what you computed.*
2. *Do not consult other sources, people, web sites, etc. If you have a question about this exam, ask the professor.*
3. *Read the questions carefully. Check your work.*
4. *Write your solutions neatly on other paper. Remember the pledge on this page, and hand in this page in with your solutions.*

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1. (8 points) Let X_1 and X_2 be independent rvs. The density of X_1 is given by

$$f_{X_1}(x_1) = \begin{cases} 2x_1 & \text{for } 0 \le x_1 \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

   X_2 is uniformly distributed on the interval $[0, 2]$. Let $Y_1 = 2X_1 + X_2$ and $Y_2 = X_1 - X_2$. Find the joint pdf of Y_1 and Y_2. Then show that you have found a nonnegative function that integrates to 1 on the appropriate domain.

2. (8 points) A fair coin is tossed until heads appears 40 times. Let X be the number of tosses required.

   (a) What is the distribution of X? State the values of any parameters necessary to specify this distribution. Use this distribution to find $P(X \le 70)$.

   (b) What continuous distribution can be used to approximate the distribution of X? State the values of any parameters necessary to specify this distribution. Use this distribution to approximate $P(X \le 70)$.

3. (6 points) Let $X_1, X_2, \ldots, X_5$ be iid random variables with cdf

$$F(x) = \begin{cases} 1 - e^{-x^2} & \text{if } x \ge 0, \\ 0 & \text{otherwise.} \end{cases}$$

   What is the expected value of the sample median?

St. Olaf Honor Pledge: I pledge my honor that on this examination I have neither given nor received assistance not explicitly approved by the professor and that I have seen no dishonest work.

Signed: _____

☐ I have intentionally not signed the pledge. (Check the box if appropriate.)