Math 262

Section 3.1 Day 19

1. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} x & 0 \le x \le 1, \\ 2 - x & 1 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Sketch the pdf of X. *If you are sitting next to a wall, sketch it on the wall!*

   (b) Find the cdf of X and sketch it (*on a wall, if you are next to one*).

   (c) What is $P(X < 1.5)$?

   (d) Find a value $\eta_{0.75}$ such that $P(X \le \eta_{0.75}) = 0.75$.

2. Suppose that a continuous random variable X has pdf $f(x) = kx(4 - x)$ for $0 \le x \le 4$, and $f(x) = 0$ otherwise.

   (a) Sketch this pdf (*on a wall, if you are next to one*). Then, without computing anything, sketch the cdf of X (make your best guess of what this looks like).

   (b) What is the value of k?

   (c) Find $P(X > 3 \text{ or } X < 1)$.

3. Suppose that the cdf of a random variable X is $F(x) = 1 - e^{-5x}$ for $x > 0$, and $F(x) = 0$ otherwise.

   (a) What is the pdf of X? Sketch both the pdf and the cdf (*on a wall...*).

   (b) What is $P\left(\frac{1}{4} < X < \frac{1}{3}\right)$? Can you get this from *either* the cdf or the pdf?

4. Random variable X has pdf

$$f(x) = \begin{cases} ax + bx^2 & 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

   Furthermore, $P(X < \frac{1}{2}) = \frac{3}{16}$. What is the median of X?

5. Let Y be a random variable with pdf given by $f(y) = \begin{cases} \frac{y}{2} & \text{if } 0 \leq y \leq 2, \\ 0 & \text{otherwise.} \end{cases}$

   (a) Find a value $\eta_{0.25}$ such that $P(Y \leq \eta_{0.25}) = 0.25$.

   (b) What is the median of Y?