Math 262

Sections 1.5 and 1.6 Day 5

1. Consider an urn containing four balls, labeled 110, 101, 011, and 000. One ball is drawn at random. For $k = 1, 2, 3$, let A_k be the event that the k^{th} digit is a 1 on the ball that is drawn.

   (a) Are the events A_1, A_2, and A_3 pairwise independent? Why or why not?

   (b) Are the events A_1, A_2, and A_3 mutually independent? Why or why not?

2. Create an example of three events A, B, C such that $P(A \cap B \cap C) = P(A)P(B)P(C)$ but the events are not mutually independent. (One way to do this is to draw a Venn diagram, specifying probabilities of A, B, C and their intersections.)

3. Suppose you flip two unfair coins: one lands heads with probability 0.4, the other lands heads with probability 0.6. What is the probability that both land heads?
   (a) Working with your group, make a plan for using a simulation in R or Mathematica to estimate this probability. Sketch your plan on the wall or on paper.
   (b) Implement your plan in R or Mathematica. Run your code to estimate the probability.
   (c) How does your simulated result compare with the exact probability that the two coins land heads?

4. Suppose you roll three standard, fair dice. What is the probability that at least two sixes appear on the dice?
   (a) Working with your group, make a plan for using a simulation in R or Mathematica to estimate this probability. Sketch your plan on the wall or on paper.
   (b) Implement your plan in R or Mathematica. Run your code to estimate the probability.
   (c) How does your simulated result compare with the exact probability that at least two sixes appear on the dice?

5. Suppose there are 3000 students at St. Olaf College. Make a plan for estimating the probability that at least 18 students share the same birthday. Then implement your plan in code and estimate the probability.