

3. Suppose that a machine produces items, 2% of which are defective. Let X be the number of defective items among 500 randomly-selected items produced by the machine.

(a) What is the distribution of X ?

(b) What are the mean and variance of X ?

(c) What is $P(X = 12)$?

(d) What Poisson distribution approximates the distribution of X ?

(e) Use your Poisson distribution to approximate $P(X = 12)$.

4. Let $X \sim \text{Poisson}(\mu)$. Show that $P(X = k)$ increases monotonically and then decreases monotonically as k increases, reaching its maximum when k is the largest integer less than or equal to μ .

Hint: Consider $\frac{P(X = k)}{P(X = k - 1)}$.