Math 262

Section 2.6 Day 11

1. Suppose that in a batch of 20 items, 3 are defective. If 5 of the items are sampled at random:

   (a) What is the probability that none of the sampled items are defective?

   (b) What is the probability that exactly 1 of the sampled items is defective?

   (c) What is the probability that exactly 4 of the sampled items are defective?

   (d) On average, how many defective items will be found in a random sample of 5 items?

   (e) What is the probability that the number of defective items sampled is within 2 standard deviations of the mean number?

2. Let X be a hypergeometric random variable with parameters n, M, N. Let Y be a Binomial random variable with parameters n and $p = \frac{M}{N}$. How does $E(X)$ compare to $E(Y)$? How does $\text{Var}(X)$ compare to $\text{Var}(Y)$?

3. Urn 1 contains 100 balls, 10 of which are red. Let X_1 be the number of red balls in a random sample of size 50 from Urn 1. Urn 2 contains 100 balls, 50 of which are red. Let X_2 be the number of red balls in a random sample of size 10 from Urn 2.

   (a) Use technology to compute the pmf of X_1. Display the values as a list or a table. Then do the same for the pmf of X_2. What do you notice?

   (b) Change the numbers 100, 10, and 50 in this problem and recompute the pmfs of X_1 and X_2. What do you notice?

   (c) Make a conjecture about when two hypergeometric random variables have the same pmf.
   BONUS: Prove your conjecture.

4. Suppose that 45% of the phone calls you receive are scam calls. Assume that the probability of a scam call is independent from one call to the next. Let X be the number of calls that you receive until (and including) the next scam call.

   (a) What is $P(X = 3)$?

   (b) If n is any positive integer, what is $P(X = n)$?

   (c) What is $E(X)$?

5. Let Y be the number of calls until (and including) the fourth scam call.

   (a) What is $P(Y = n)$?

   (b) What is $E(Y)$?

6. An interviewer must find 10 people who agree to be interviewed. Suppose that when asked, a person agrees to be interviewed with probability 0.4.

   (a) What is the probability that the interviewer will have to ask exactly 20 people?

   (b) What are the mean and standard deviation of the number of people that the interviewer will have to ask?

7. If X has a geometric distribution with parameter p, and k is a positive integer, what is $P(X > k)$?

8. Scam calls, again. Suppose that 45% of the phone calls you receive are scam calls.

   (a) What is the probability that *none* of the first 4 calls are scam calls?

   (b) If none of the first 4 calls are scam calls, what is the probability that none of the first 7 calls are scam calls?

   (c) If none if the first 4 calls are scam calls, what is the probability that none of the first $4 + k$ calls are scam calls?

★ **BONUS:** Show that $\sum_{k=1}^{\infty} k(1-p)^{k-1}p = \frac{1}{p}$. This proves that the mean of a geometric random variable is $\frac{1}{p}$. *Hint*: Start with a geometric series. What is its sum? Then differentiate.