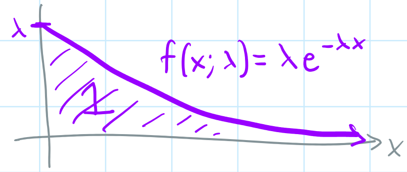
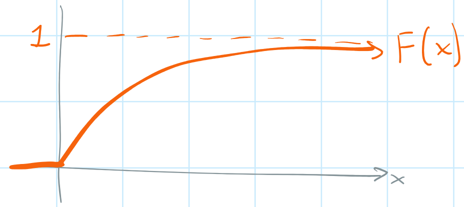
EXPONENTIAL DISTRIBUTION

The times between events in a Poisson process are exponentially distributed.

- **pdf:** $f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x} & x>0 \\ 0 & \text{otherwise.} \end{cases}$



- **cdf:** $F(x) = \begin{cases} 1-e^{-\lambda x} & \text{for } x>0 \\ 0 & \text{otherwise.} \end{cases}$



- **mean** $E(X) = \frac{1}{\lambda}$ $\lambda =$ "rate"

- **variance:** $Var(X) = \frac{1}{\lambda^2}$ $\sigma_X = \frac{1}{\lambda}$

③ exponential mgf

$X \sim Exp(\lambda)$

$$M_X(t) = E(e^{tX}) = \int_0^\infty \underbrace{e^{tx}}_{\text{values}} \cdot \underbrace{\lambda e^{-\lambda x}}_{\text{density}} \, dx = \lambda \int_0^\infty e^{tx} e^{-\lambda x} \, dx$$

$$= \lambda \int_0^\infty e^{tx-\lambda x} dx = \lambda \int_0^\infty e^{x(t-\lambda)} dx = \frac{\lambda}{t-\lambda} e^{x(t-\lambda)} \Big|_{x=0}^{\infty}$$

Need: $t-\lambda<0$ so integral converges $t<\lambda$

$$= 0 - \frac{\lambda}{t-\lambda} e^{0(t-\lambda)} = \frac{-\lambda}{t-\lambda} = \frac{\lambda}{\lambda-t}$$

$$\boxed{M_X(t) = \frac{\lambda}{\lambda - t} \quad \text{for} \quad \underline{t<\lambda}}$$

④ $\lambda = 1$:

$$M_X(t) = \frac{1}{1-t} = (1-t)^{-1}$$

$$M_X'(t) = -1(1-t)^{-2}(-1) = (1-t)^{-2}$$

$$M_X'(0) = (1-0)^{-2} = 1 = E(X)$$

$$E(X^2),\ E(X^3), \ldots$$