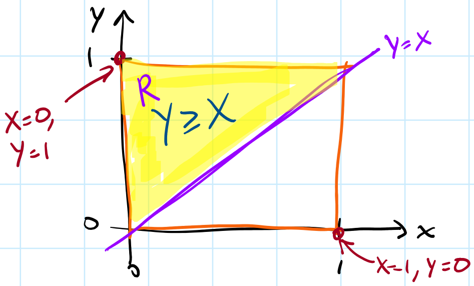
From last time

#2 X and Y uniformly distributed on a unit square



joint density: $f(x,y) = \begin{cases} 1 & \text{if } 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$

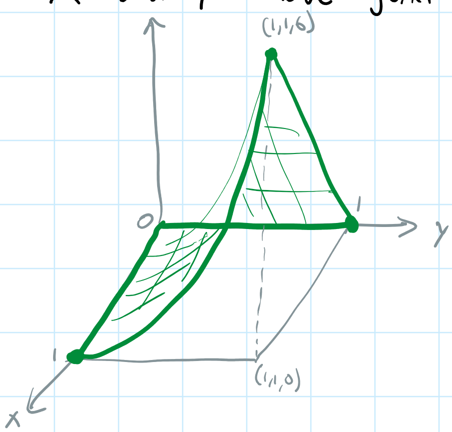
c) $P(X \le Y) = \iint_R f(x,y)\,dA = \iint_R 1\,dA = \text{Area}(R) = \boxed{\frac{1}{2}}$

first consider $X = Y$

d) X and Y are independent: $f(x,y) = f_X(x) f_Y(y)$

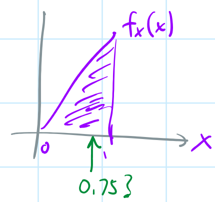
$$1 = 1 \cdot 1$$

1) X and Y have joint pdf $f(x,y) = 6xy^2$ for $0 \le x \le 1$, $0 \le y \le 1$

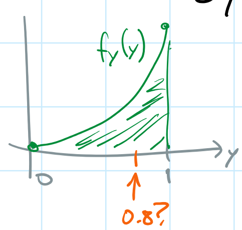


Marginal pdfs:

$$f_X(x) = \int_0^1 6xy^2\,dy = 2x \quad \text{for } 0 \le x \le 1$$



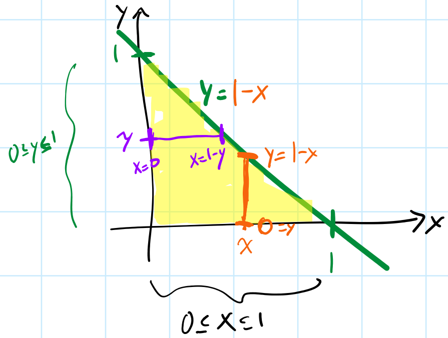
$$f_Y(y) = \int_0^1 6xy^2\,dx = 3y^2 \quad \text{for } 0 \le y \le 1$$



② $0 \le x, \quad 0 \le y, \quad x + y \le 1$

$\to x + y = 1$

$y = 1 - x$



$$\int_0^1\int_0^{1-x} (3x+3y)\, dy\, dx = 1$$

$$\int_0^1\int_0^{1-y} (3x+3y)\, dx\, dy = 1$$

Expected Value:

$$E\big(h(X,Y)\big) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)\, f(x,y)\, dy\, dx$$

Covariance:

$$\mathrm{Cov}(X,Y) = E\big[(X-\mu_X)(Y-\mu_Y)\big] = E(XY) - E(X)\,E(Y)$$

Correlation:

$$\mathrm{Corr}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sigma_X \cdot \sigma_Y}$$