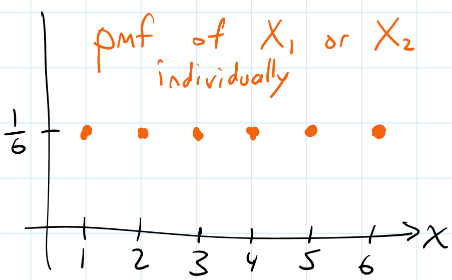
DENSITY OF A SUM

Let X and Y be independent random variables and $W = X + Y$.

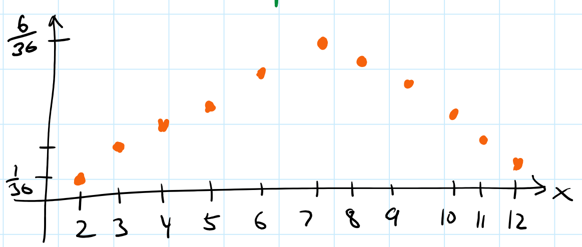
For continuous variables: $$\underbrace{f_W(w) = \int_{-\infty}^{\infty} f_X(x)\, f_Y(w-x)\, dx}_{\text{convolution integral}}$$

For all variables: $$M_W(t) = M_X(t)\, M_Y(t) \quad \leftarrow \text{mgfs}$$

From last time: X_1 and X_2 were rolls of 2 standard 6-sided dice



What is the pmf of $X_1 + X_2$?



	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

③ $X_k \sim N(k, 1)$ has mgf $M_{X_k}(t) = e^{kt + 1^2 t^2/2} = \exp\left(kt + \frac{t^2}{2}\right)$

$\mu = k$
$\sigma = 1$

mgf of $X_1 + X_2 + \cdots + X_m$:

$e^A \cdot e^B = e^{A+B}$

$$\underset{k=1}{M_{X_1}(t)} \cdot \underset{k=2}{M_{X_2}(t)} \cdots \underset{k=m}{M_{X_m}(t)}$$

$$= \exp\left(1t + \frac{t^2}{2}\right) \cdot \exp\left(2t + \frac{t^2}{2}\right) \cdot \exp\left(3t + \frac{t^2}{2}\right) \cdots \exp\left(mt + \frac{t^2}{2}\right)$$

$$= \exp\left(1t + \frac{t^2}{2} + 2t + \frac{t^2}{2} + 3t + \frac{t^2}{2} + \cdots + mt + \frac{t^2}{2}\right)$$

$$= \exp\left((1+2+3+\cdots+m)t + m\frac{t^2}{2}\right)$$

this is the mgf of a Normal distribution with mean $\mu = 1+2+3+\cdots+m$ and $\sigma^2 = m$

Thus, $X_1 + X_2 + \cdots + X_m \sim N(1+2+\cdots+m, m)$.