Hypergeometric Distribution

A set contains N items, M of which are "successes" and the rest are "failures." A sample of n items is selected without replacement (each subset of size n is equally likely to be chosen). Let X be the number of successes in the sample. Then $X \sim \text{Hypergeometric}(n, M, N)$.

pmf: $$P(X = x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

QUESTION: What values of x have nonzero probability?

- Certainly $0 \le x$
- Num. successes bound: $x \le M$
- Sample size bound: $x \le n$

$\Big\}$ $x \le \min(n, M)$

- Bound on num. of failures in the sample:

$$n - x \le N - M$$

$$n - N + M \le x \longrightarrow \max(0, n-N+M) \le x$$

Book says: $\max(0, n-N+M) \le x \le \min(n, M)$