Poisson Process: A sequence of discrete occurrences such that the average number of occurrences in a fixed time interval is known, but the exact timing of the occurrences is random.

Examples: arrival of emails
people that enter a store
arrival of taxis at a taxi stand
homework problems per day?
emission of alpha particles from a radioactive substance

Poisson Distribution: $X \sim \text{Poisson}(\mu)$ if X counts the occurrences of a Poisson process in a time interval with mean μ occurrences per time interval.

pmf

$$P(X=x) = p(x;\mu) = e^{-\mu}\frac{\mu^x}{x!} \quad \text{for } x \in \{0,1,2,3,\dots\}$$

check: $p(x;\mu) > 0$ yes

Sum to 1?

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\sum_{x=0}^{\infty} p(x;\mu) = \sum_{x=0}^{\infty} e^{-\mu}\frac{\mu^x}{x!} = e^{-\mu}\sum_{x=0}^{\infty}\frac{\mu^x}{x!} = e^{-\mu}\sum_{k=0}^{\infty}\frac{\mu^k}{k!}$$

$$= e^{-\mu}\left(e^{\mu}\right) = 1$$