MATH 262

Section 4.1 Day 28

1. A cafeteria has three meal options: pizza, burgers, and salad bar. Three students each choose one option independently at random (equally likely to choose any option). Let X be the number (of the 3) who choose pizza, and let Y be the number who choose the salad bar.

   (a) *Discuss with your neighbors:* How can you compute the following probabilities?

   $P(X = 0 \text{ and } Y = 0)$

   $P(X = 0 \text{ and } Y = 1)$

   (b) Generalize part (a) to other values of X and Y. Make a table showing the joint pmf of X and Y.

   (c) What are the marginal pmfs of X and Y?

   (d) *Discuss with your neighbors:* Are X and Y independent? Why or why not?

2. Suppose a particle is randomly located in the unit square $0 \leq x \leq 1$, $0 \leq y \leq 1$. Let (X, Y) be the coordinates of the particle.

   (a) *Discuss with your neighbors:* Is $f(x, y) = 1$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$ a reasonable joint density function of X and Y? Why or why not?

   (b) Find $P(X \leq 0.2, 0.1 \leq Y \leq 0.5)$.

   (c) Find $P(X \leq Y)$.

   (d) Are X and Y independent? Why or why not?

3. Let X and Y have joint pdf $f(x, y) = 6xy^2$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

   (a) Verify that $f(x, y)$ is a joint pdf.
   (b) What is $f_X(x)$?
   (c) What is $P(X \leq Y)$?
   (d) Are X and Y independent? Why or why not?