

MATH 262

Section 4.3

Day 30

1. Let $X \sim \text{Unif}[-1, 1]$ and $Y = X^2$.

(a) Compute the following:

$$E(X)$$

$$E(Y)$$

$$E(XY)$$

(b) *Discuss with your neighbors:* Are X and Y independent? Are X and Y correlated? Why or why not?

The rest of this worksheet involves the following two scenarios:

I. Two standard, fair dice are rolled. Let X_1 and X_2 be the numbers that appear on the dice.

II. An urn contains balls labeled 1, 2, 3, 4, 5, 6. Let Y_1 and Y_2 be the numbers on two balls drawn without replacement from the urn.

2. *Discuss with your neighbors:* What is the distribution of X_1 and X_2 ? How about the distribution of Y_1 and Y_2 ?

3. What are the expected values of X_1 , X_2 , Y_1 , and Y_2 ? What are the variances of these random variables?

4. What are $E(X_1 + X_2)$ and $\text{Var}(X_1 + X_2)$?

5. What are $E(Y_1 + Y_2)$ and $\text{Var}(Y_1 + Y_2)$?

Hint: $E(Y_1 Y_2)$

6. Sketch the pmfs of $X_1 + X_2$ and $Y_1 + Y_2$. How does this help make sense of the means and variances that you found for these sums?

7. Generalize to rolls of n dice: find $E(X_1 + \cdots + X_n)$ and $\text{Var}(X_1 + \cdots + X_n)$.

★ **BONUS:** Generalize to choosing n balls from the urn. Find $E(Y_1 + \cdots + Y_n)$ and $\text{Var}(Y_1 + \cdots + Y_n)$.