MATH 262

Sections 4.3.1–4.3.2 Day 31

1. Let X and Y be independent uniform variables on $[0, 1]$, and let $W = X + Y$.

   (a) *Discuss with your neighbors:* What do you think the pdf of W will look like? Make a guess. Draw a sketch.

   (b) Write down a convolution integral formula for the pdf of W.

   (c) *Discuss with your neighbors:* How do you actually compute the integral you wrote down in (b)? *Hint:* Consider the cases $w \leq 1$ and $w \geq 1$ separately.

2. Use convolution to write an integral that gives the pdf of the sum of three independent Unif$[0, 1]$ random variables. How could you evaluate this integral?

3. Let $X_k \sim N(k,1)$ for $k \in \{1,2,\dots,m\}$, and suppose all of the X_k are independent.

(a) What is the distribution of $X_1 + X_2 + \cdots + X_m$?

(b) What is the distribution of $X_1 + 2X_2 + 3X_3 + \cdots + mX_m$?

4. Use moment generating functions to justify the following statements.

(a) The sum of n independent exponential random variables with common parameter λ has a gamma distribution with parameters $\alpha = n$ and $\beta = 1/\lambda$.

(b) The sum of n independent geometric random variables with common parameter p has a negative binomial distribution with parameters $r = n$ and p.

mgf reference:

Normal: $e^{\mu t + \sigma^2 t^2/2}$ Exponential: $\frac{\lambda}{\lambda - t}$ Gamma: $\left(\frac{1}{1-\beta t}\right)^{\alpha}$

Geometric: $\frac{pe^t}{1-(1-p)e^t}$ Negative Binomial: $\left(\frac{pe^t}{1-(1-p)e^t}\right)^{r}$