SYMMETRY OF FUNCTIONS

ODD: $f(-x) = -f(x)$, graph has rotational symmetry about the origin

EVEN: $f(-x) = f(x)$, graph is symmetric about the y-axis

If $f(x)$ is an odd function, then its Fourier series has no cosine terms.

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{L}x\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L}x\right)$$ **FOURIER SERIES FOR f**

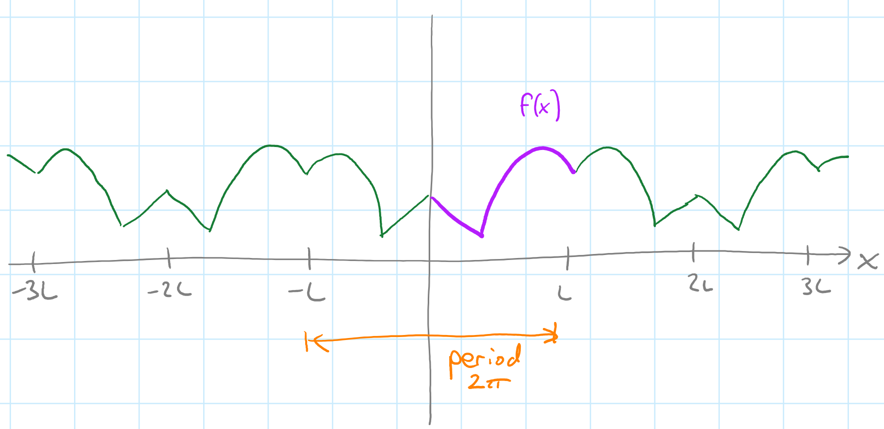
$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx, \qquad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right)dx \qquad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi}{L}x\right)dx$$

If $f(x)$ is odd, then a_0 and a_n are zero (integrals of odd functions on $-L<x<L$ are zero, by symmetry.)

If $f(x)$ is even, then b_n is zero.

If $f(x)$ is an even function: $f(-x) = f(x)$, then its Fourier series has no sine terms.

FOURIER COSINE SERIES of $f(x)$ on $0 \le x \le L$ is the Fourier series of the even periodic extension of f (with period $2L$).



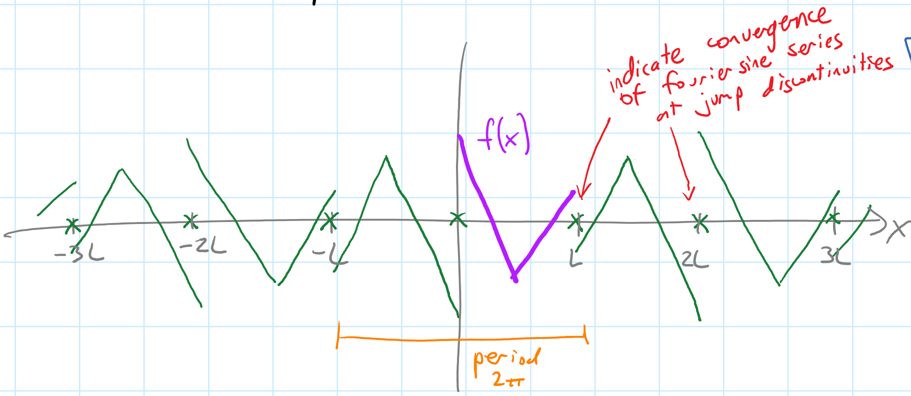
FOURIER COSINE SERIES

$$f(x) \sim A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi}{L}x\right)$$

$$A_0 = \frac{1}{L}\int_0^L f(x)\,dx$$

$$A_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi}{L}x\right)dx$$

FOURIER SINE SERIES of $f(x)$ on $0 \le x \le L$ is the Fourier series of the odd periodic extension of f (with period $2L$).



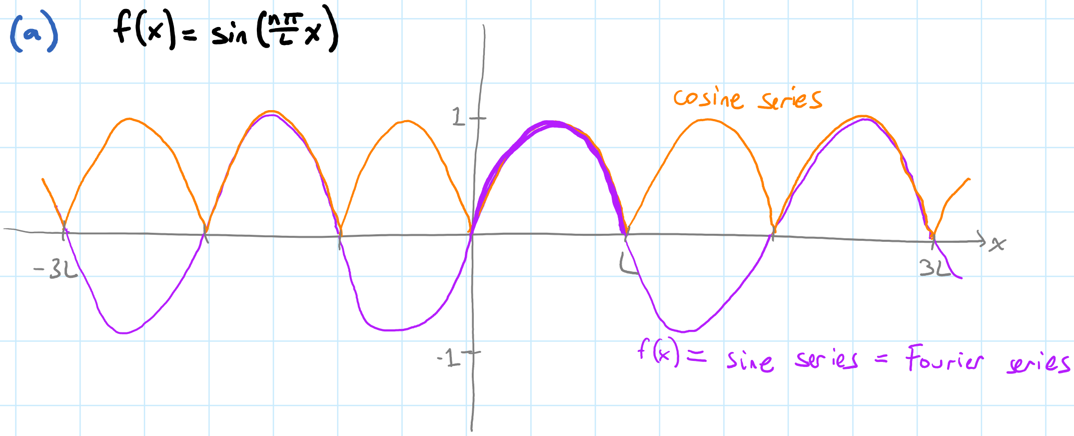
FOURIER SINE SERIES

$$f(x) \sim \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi}{L}x\right)$$

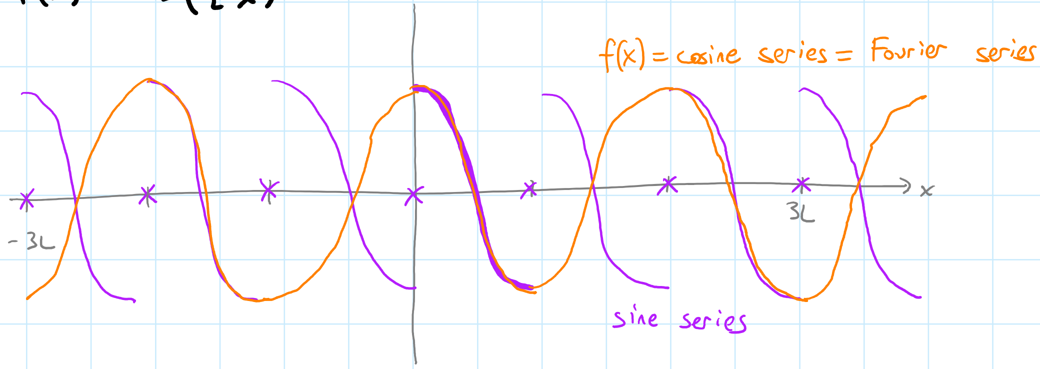
$$B_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx$$

WORKSHEET: FOURIER SINE AND COSINE SERIES

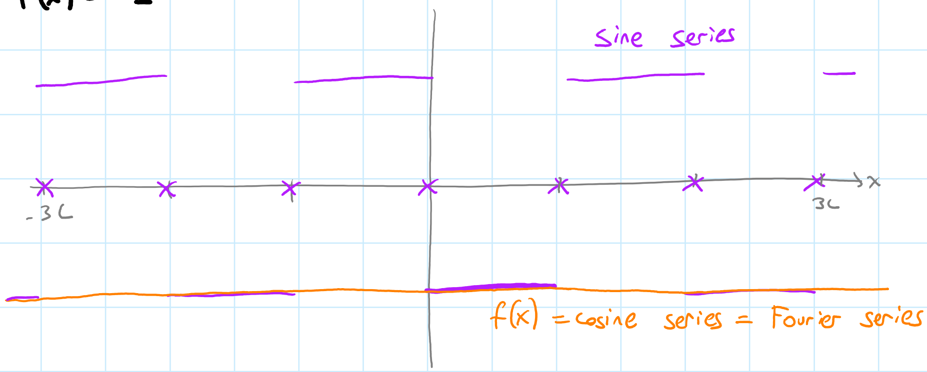
1. (a) $f(x) = \sin\left(\frac{n\pi}{L}x\right)$



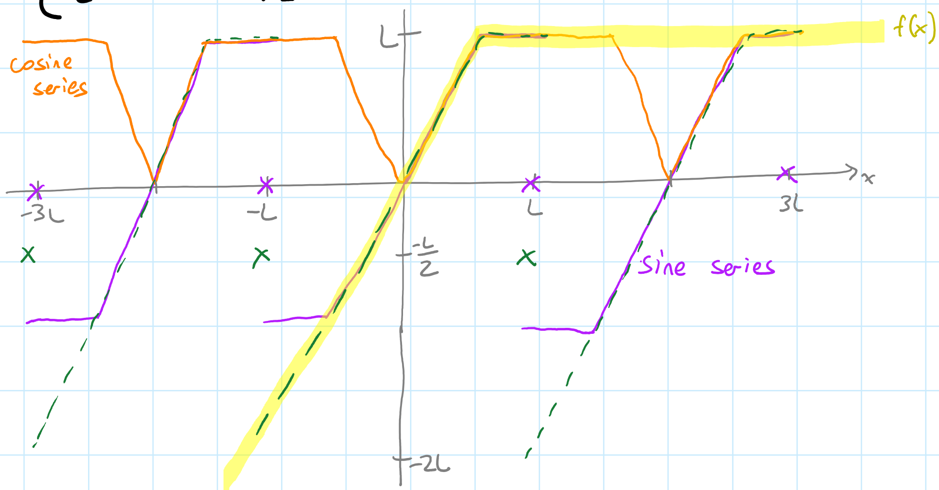
(b) $f(x) = \cos\left(\frac{n\pi}{L}x\right)$



(c) $f(x) = -1$



(d) $f(x) = \begin{cases} 2x & \text{if } x < L/2 \\ L & \text{if } x > L/2 \end{cases}$



2. (a) Fourier series for $f(x)$ is continuous and converges pointwise to $f(x)$ iff the periodic extension of $f(x)$ is continuous (Convergence theorem)

That is: $f(-L) = f(L)$ and f is continuous on $-L \le x \le L$

(b) Cosine series is continuous and converges to f iff:

$f(x)$ is continuous

(c) Sine series is continuous and converges to f iff:

$f(x)$ is continuous and $f(0) = f(L) = 0$.