Math 330 Reading Questions

Section 1.5 ______________________ NAME

First, some concepts from multivariable calculus.

For a function $f(x, y, z) : \mathbb{R}^3 \to \mathbb{R}$, the **gradient** of f, denoted ∇f is the vector of partial derivatives of f,

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle.$$

At any point, the gradient is a vector that points in the direction of steepest ascent.

For a continuously differentiable vector field $F = \langle U, V, W \rangle$, the **divergence** of F, denoted $\nabla \cdot F$, is the scalar

$$\nabla \cdot F = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}.$$

The divergence measures the extent to which the vector field behaves like a source at a given point.

The **divergence theorem** says that *the outward flux through a closed surface equals the volume integral of the divergence inside the surface.* Formally, if R is a closed, bounded region in $\mathbb{R}^3$ with piecewise smooth boundary S, and F is a vector field as before, then

$$\iiint_R (\nabla \cdot F)\, dV = \oiint_S (F \cdot \hat{\mathbf{n}})\, dS$$

where $\hat{\mathbf{n}}$ is the outward normal unit vector to surface S. The triple integral on the left gives the total contribution of sources and sinks within R. The surface integral on the right gives the total flux $F \cdot \hat{\mathbf{n}}$ through surface S. Note that the divergence theorem is a higher-dimensional analog of the fundamental theorem of calculus.

Now, the reading questions.

Answer the following questions as you read ection 1.5 in the textbook. This sheet will be collected at the beginning of class on Tuesday. Your answers will be graded for completeness.

1. What integral gives the total heat energy in subregion R?

2. What integral gives the total heat energy flowing out of region R per unit time?

3. What integral gives the total heat energy generated per unit time in region R?

4. What role does the divergence theorem play in the derivation of the multidimensional heat equation?

5. What is the multidimensional version of Fourier's law of heat conduction?

6. State at least one version of the multidimensional heat equation.

7. What is the **Laplacian** of a function $u(x, y, z)$? How is the Laplacian expressed in terms of the partial derivatives of U?