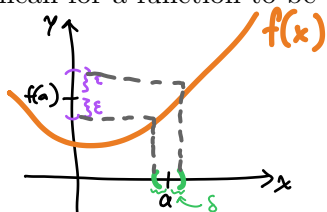


Continuity

MATH 348

Class notes
September 5, 2024

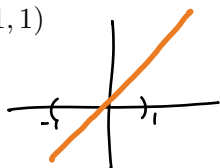
1. What does it mean for a function to be **continuous**?



2. Which of the following functions are continuous on the specified domain?

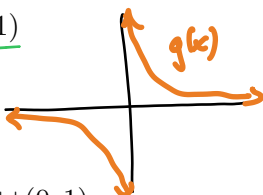
(a) $f(x) = x$ on $(-1, 1)$

Yes



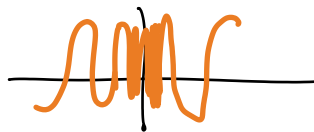
(b) $g(x) = \frac{1}{x}$ on $(-1, 0) \cup (0, 1)$

Yes



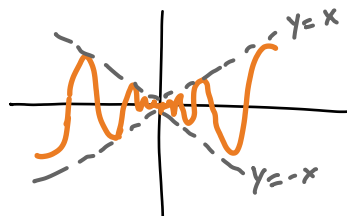
Note: $x=0$ is not in the domain

(c) $h(x) = \sin\left(\frac{1}{x}\right)$ on $(-1, 0) \cup (0, 1)$



Yes, on the domain
Not possible to define $h(0)$ so that h is continuous on $(-1, 1)$.

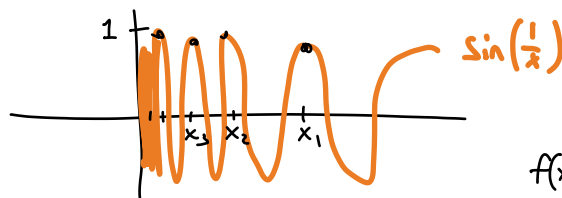
(d) $k(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ on $(-1, 1)$



Yes.

3. If the sequence x_1, x_2, x_3, \dots converges to some value a and the sequence $f(x_1), f(x_2), f(x_3), \dots$ converges to $f(a)$, does this mean that function f is continuous at a ? Why or why not?

No.



$$x_1, x_2, x_3, \dots \rightarrow 0$$

$$f(x_1) = f(x_2) = f(x_3) = \dots = 1$$

4. If function f has the property that for every sequence x_1, x_2, x_3, \dots that converges to a , the sequence $f(x_1), f(x_2), f(x_3), \dots$ converges to $f(a)$, then is f continuous at a ? Why or why not?

Yes.

Open Sets

MATH 348

1. If function f is continuous and A is an open set, what can you say about $f(A)$?
2. If function f is continuous and A is an open set, what can you say about $f^{-1}(A)$?
3. Is the union of every collection of open sets itself an open set?
4. Is the intersection of every collection of open sets itself an open set?
5. Is the intersection of every finite collection of open sets itself an open set?