Bases

MATH 348

1. Let $\mathcal{B} = \{(a,b) \cup (c,d) \subset \mathbb{R} \mid a < b < c < d$ are distinct irrational numbers$\}$. Is $\mathcal{B}$ a basis for the standard topology on $\mathbb{R}$?

2. Let $\mathcal{B} = \{(n, n+1) \mid n \in \mathbb{Z}\}$. Is $\mathcal{B}$ a basis for some topology on $\mathbb{R}$?

3. The collection $\mathcal{B} = \{\{x\} \mid x \in \mathbb{R}\}$ is a basis for what topology on $\mathbb{R}$?

4. Let $\mathcal{B} = \{[a,b) \subset \mathbb{R} \mid a < b\}$. Is this a basis for a topology on $\mathbb{R}$? Is this topology the standard topology?

5. Give three examples of bases for the standard topology on $\mathbb{R}^2$

6. For each $n \in \mathbb{Z}$ define the set

$$B(n) = \begin{cases} \{n\} & \text{if } n \text{ is odd,} \\ \{n-1, n, n+1\} & \text{if } n \text{ is even.} \end{cases}$$

Is $\mathcal{B} = \{B(n) \mid n \in \mathbb{Z}\}$ the basis for a topology on $\mathbb{Z}$?

Give an example of a continuous function $f : \mathbb{Z} \to \mathbb{Z}$ in this topology. Then give an example of a discontinuous function.

Connectivity

MATH 348

1. Which of the following topological spaces are connected?

(a) $\mathbb{R}$ with the standard topology

(b) $\mathbb{R}$ with the discrete topology

(c) $\mathbb{R} - \{\pi\}$ with the subspace topology

(d) S^0 with the subspace topology

2. Let X be a connected topological space. Show that there is no continuous surjective map from X to S^0.

3. Let X be a disconnected topological space. Is there a continuous surjective map from X to S^0?