

Homeomorphisms

MATH 348

1. Which of the following spaces do you think we should regard as topologically *the same*? What should “topologically the same” mean?

(a) $(0, 1)$

(b) $[0, 1]$

(c) $(0, 1]$

(d) S^1

(e) \mathbb{R}

(f) $[0, 2\pi)$

(g) $\mathbb{R} \cup \{\infty\}$

(h) (a, b) for any $a < b \in \mathbb{R}$

(i) $[a, b]$ for any $a < b \in \mathbb{R}$

2. Find three different topologies on the three-point set $\{a, b, c\}$, each consisting of five open sets, such that two of the topologies are homeomorphic, but the third is not homeomorphic to the other two.

3. Let $f : S \rightarrow T$ be a homeomorphism and $g : T \rightarrow U$ be another homeomorphism. Show that $g \circ f : S \rightarrow U$ is a homeomorphism.

4. Let S and T be homeomorphic spaces. Prove each of the following:

(a) If S is connected, then so is T .

(b) If S is compact, then so is T .

(c) If S is Hausdorff, then so is T .

5. Suppose $f : X \rightarrow Y$ is a continuous bijective function. Is f^{-1} necessarily continuous?