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If S and T are topological spaces, the disjoint union $S \amalg T$ consists of all points in S together with all points in T, with all of these points regarded as distinct. Any points in $S \cap T$ are counted twice in $S \amalg T$.

A subset $Q \subset S \amalg T$ is open if $Q \cap S$ is open in S and $Q \cap T$ is open in T.

Example 1: $S = \{1, 2, 3\}$ and $T = \{2, 4\}$

then $S \amalg T = \{1, 2_S, 3, 2_T, 4\}$

Example 2: $S = \{a, b\}$ with the discrete topology

$T = \{c, d, e$

Disjoint Unions and Product Spaces

MATH 348

1. Describe each of the following spaces

(a) $\mathbb{R} \amalg \mathbb{R}$

(b) $\{-1\} \amalg \{+1\}$

(c) $\mathbb{R} \amalg S^1$

2. Let S and T be topological spaces. Prove that $S \amalg T$ is compact if and only if both S and T are both compact.

3. Under what circumstances is $S \amalg T$ a Hausdorff space? Formulate and prove a statement similar to #2 above.

4. Describe each of the following spaces

(a) $\mathbb{R} \times S^1$

(b) $\mathbb{R} \times S^0$

(c) $S^1 \times [1, 2]$

(d) $S^1 \times S^1$

5. Suppose X is a finite set of n elements and Y is a finite set of m elements.

(a) How many elements are there in $X \amalg Y$?

(b) How many elements are there in $X \times Y$?

6. For any product space $S \times T$, show that the projection maps $p_1 : S \amalg T \to S$ and $p_2 : S \amalg T \to T$ are continuous.

7. Let S and T be topological spaces. Prove that $S \times T$ is connected if and only if both S and T are both connected.