17 October 2024

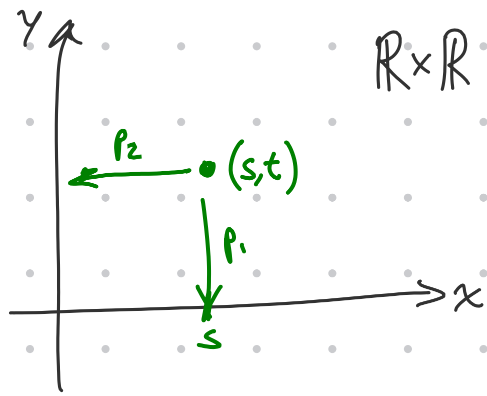
From last time: disjoint unions and products

Inclusion maps: $i_1 : S \to S \amalg T$ and $i_2 : T \to S \amalg T$

Continuous: if $U \subset S \amalg T$ is open, this means that $U \cap S$ and $U \cap T$ are open.
Then $i_1^{-1}(U) = U \cap S$ and $i_2^{-1}(U) = U \cap T$ are open, so i_1 and i_2 are continuous.

Product: projection maps: $p_1 : S \times T \to S$ and $p_2 : S \times T \to T$

$p_1(s,t) = s$ $\qquad$ $p_2(s,t) = t$



$\mathbb{R} \times \mathbb{R} \cong \mathbb{R}^2$

Quotient Spaces

MATH 348

1. Construct the torus T^2 as:

(a) A quotient of the cylinder C

(b) A quotient of $X = [0,1] \times [0,1]$

(c) A quotient of $\mathbb{R}^2$

2. What space results from identifying each pair of parallel edges of the square $[0,1] \times [0,1]$, with one edge reversed?

3. Show that if $f : X \to Y$ is a surjective continuous map that is either open or closed, then f is a quotient map.

4. Let $X = [0,1] \cup [2,3] \subset \mathbb{R}$ and $Y = [0,2] \subset \mathbb{R}$. Define $f : X \to Y$ as

$$f(x) = \begin{cases} x & \text{if } x \in [0,1] \\ x-1 & \text{if } x \in [2,3] \end{cases}.$$

Is f surjective? Continuous? Open? Closed? A quotient map?

5. Let $\pi_1 : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be projection onto the first coordinate. Is π_1 surjective? Continuous? Open? Closed? A quotient map?