Quotient Spaces

MATH 348

1. Describe each quotient space given by X/A or $X/\sim$.

(a) $X = [0, 1]$ and $A = \partial X = \{0, 1\}$

(b) $X = [0, 1] \times [0, 1]$ and $A = \partial X$

(c) $X = [0, 1]^n$ and $A = \partial X$

(d) $X = \{a, b, c, d, e\}$ with the topology $\{\varnothing, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}, X\}$. Define an equivalence relation on X by $a \sim b$ and $c \sim d \sim e$.

(e) $X = [0, 1]$ with $x \sim x'$ if $x - x' \in \mathbb{Z}$

(f) $X = [0, 1]^2$ with $(x, y) \sim (x', y')$ if $x - x' \in \mathbb{Z}$ and $y - y' \in \mathbb{Z}$

(g) $X = [0, 1]^3$ with $(x, y, z) \sim (x', y', z')$ if $x - x' \in \mathbb{Z}$, $y - y' \in \mathbb{Z}$, and $z - z' \in \mathbb{Z}$

(h) $X = [0, 1]^2$ with each side identified with the opposite side with directions reversed

2. Suppose X is a topological space with equivalence relation $\sim$. Let Q be the quotient space $X/\sim$, and let $\pi : X \to Q$ be the projection (quotient) map.

(a) If S is any other topological space and $f : Q \to S$ is continuous, then there exists a map $g : X \to S$ such that the following diagram commutes.

(b) If $g : X \to S$ is any continuous function such that $g(x) = g(y)$ whenever $x \sim y$, then there is a continuous function $f : Q \to S$ such that the following diagram commutes.