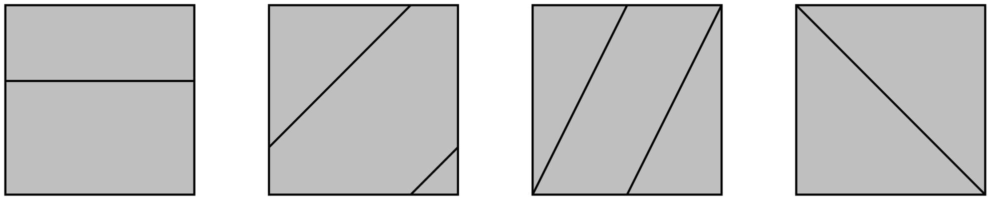
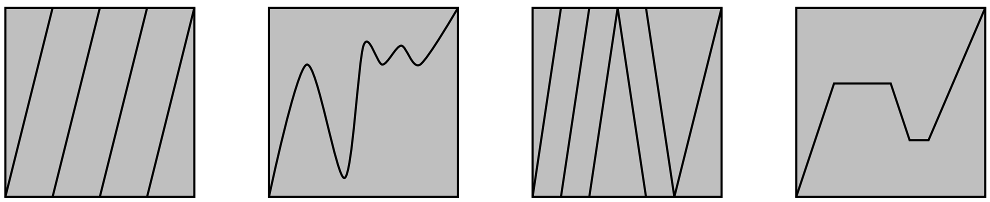
Circle Maps

MATH 348

1. What circle maps are shown in the following graphs?



2. What is the **degree** of each of the circle maps graphed above? How about each of the following?



3. Sketch the graph of a circle map with degree 5. Then sketch a circle map with degree -2.

4. Prove the statement: If a circle map $f : S^1 \to S^1$ has a nonzero degree, then it is surjective.

5. Under what condition, do you think, are two circle maps are homotopic? Why?

6. Is the circle S^1 contractible? Can you prove your answer?

2D Brouwer's Fixed-Point Theorem

MATH 348

2-D Brouwer's Fixed-Point Theorem: Let $D^2 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$ be the closed disk, and let $f : D^2 \to D^2$ be a continuous map. Then there is some point $(x, y) \in D^2$ such that $f(x, y) = (x, y)$.

Complete the following steps to prove Brouwer's fixed-point theorem in 2-D:

(a) Suppose $f : D^2 \to D^2$ does not have a fixed point. Explain why this means that we can draw a line through $f(x, y)$ and (x, y) and extend this line through (x, y) until it meets the boundary of D^2.

(b) Define a function $g : D^2 \to S^1$ such that $g(x, y)$ is the point where the line from $f(x, y)$ through (x, y) meets the boundary of D^2. Explain why g is continuous.

(c) Define a map

$$F : S^1 \times I \to S^1$$

by $F((x, y), t) = g(tx, ty)$. Argue that F is a homotopy between maps $h(x, y) = F((x, y), 0)$ and $j(x, y) = F((x, y), 1)$. Describe the maps f and j.

(d) What are the degrees of the maps h and j? How does this produce a contradiction?