Topology Term Paper

MATH 348

The topology term paper is your chance to explore a topological topic of interest to you. Working individually or in a group of up to three people, you will research a topic and write an expository paper. Your research is not expected to be original or novel, however, it is expected that you gain familiarity with the literature on your topic (properly citing that work) and convey your findings clearly and precisely.

Possible Topics

Some possible topics appear below. This list is not intended to be exhaustive—feel free to come up with other ideas as well!

When selecting a topic, make sure that you can find multiple sources of information for your topic. Your sources should primarily be academic papers and books written at a level that you can understand. Your paper should not be primarily a re-write of a single research article, but should distill insights from multiple sources.

1. **Metric spaces**: Compare and contrast metric spaces and topological spaces, and explore the different properties that arise. Possible applications include error-correcting codes, the Hamming distance, and DNA sequence comparisons.
2. **Separation axioms in topology**: Explore the hierarchy of Tychonoff separation axioms and their implications in topological spaces.
3. **Topological phenotype spaces:** Explore how topology can help us understand the evolution of biological traits.
4. **Geographic information systems:** Explore how topological concepts quantify spatial relationships between neighboring geographic features.
5. **Topological manifolds and cosmology:** What is the shape of the universe? Study how topology may shed light on this question.
6. **Knot theory:** Mathematical knots are loops in 3-dimensional space. Study ways in which topology can determine whether two knots are the same or different.
7. **The Brouwer fixed point theorem:** Provide an overview of the Brouwer fixed-point theorem, including its proof and applications. Discuss how the theorem has implications for various fields like economics and physics.
8. **Digital image topology:** Explore applications of topology to digital images and digital image processing.
9. **Configuration spaces:** A configuration space encodes all possible positions of object such as a robot. These can be studied topologically and applied to solve motion-planning and navigation problems.
10. **Topological dynamics:** Explore how continuous transformations on topological spaces can help us understand dynamical systems.

11. **Topological properties of fractals:** Explore the topological properties of fractals, such as dimension, connectedness, and self-similarity.

12. **Topological networks:** Topology can be applied to the study of networks, such as biological networks or brain networks.

13. **Differential topology:** Give an introduction to differential topology, which studies differentiable functions on differentiable manifolds, connecting with geometry and analysis.

14. **Structure of proteins:** Explore how topology can help us understand the structure of protein molecules. This could include protein folding or knotted molecules.

15. **Pick your own topic!** Choose your own topological concept or application to investigate.

Timeline

The following is a schedule for work on the projects.

- **End of October:** Think about what you might want to research for your project. Identify three possible topics. Discuss your ideas with your classmates and think about who you would like to work with.
- **Thursday, November 5:** Complete the Term Paper Planning Survey. This will ask you for possible topics and who you do (or don't) want to work with.
- **Tuesday, November 10:** Teams and topics finalized.
- **Thursday, November 19:** List of at least three sources due.
- **Thursday, December 3:** Outline of paper due.
- **Tuesday, December 8:** Draft of paper due.
- **Tuesday, December 15:** Final paper due.

Paper

Your paper should be written in a professional mathematical style and typed in LaTeX. Imagine that your audience is students with similar mathematical knowledge as you, but who are not in this course. All illustrations must be of professional quality with no handwritten elements. Illustrations must be numbered consecutively and cited in the text. Please make sure all figures are legible with large enough font and clearly labeled with a descriptive caption.

For some guidelines about writing a math paper, consult *Guidelines for Good Mathematical Writing* by Francis Su[1] and *How to Write Mathematical Papers* by Bruce Berndt.[2]

The content of your paper should include as much of the following as is reasonable:

- Precise description of the concept, method, or application that you studied.
- History of your topic and important publications in which it appears.

[1]Francis Edward Su, *Guidelines for Good Mathematical Writing*, https://scholarship.claremont.edu/cgi/viewcontent.cgi?article=2154&context=hmc_fac_pub

[2]Bruce Berndt, *How to Write Mathematical Papers*, https://alozano.clas.uconn.edu/wp-content/uploads/sites/490/2020/08/berndt.pdf.

- Relevant definitions, proofs, and examples.

The recommended length of your paper is 8–15 pages, though lengths vary depending on the topic, the size of your group, inclusion of diagrams of diagrams in your paper, etc. It's more important to write a quality paper than to meet a particular page length.

- The title of your paper should be brief and should describe the content of your paper.
- An abstract not exceeding 200 words that summarizes the principal concepts and conclusions of your work must appear following your title, but before the content of your paper. Format your abstract using an abstract environment in LaTeX. (The abstract is not Section 1 of the paper.)
- You must use mathematical terminology correctly (especially terminology from class).
- You must use mathematical notation correctly (especially notation from class).
- Examples and diagrams as appropriate to illustrate important concepts, especially technical concepts.
- Your writing should be of professional quality. This includes proper grammar, good sentence structure, and thoughtful organization using paragraphs and section headings.
- Your writing style should convey attention to detail.
- It's OK to focus on applications, but the paper can't just be about physics or biology. It's essential to connect topology to these applications; for example, show how topological methods can lead to scientific insights.
- Illustrations should be professional quality, not hand-drawn. It's OK to borrow diagrams from other figures if you cite your sources.